Final Revision

Algebra & Trig

Sec 1

Math

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Remember that (Algebra):

- Solving Quadratic Equations: $ax^2 + b\,x + c = 0$

  By the general formula: $X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Vertex of Quadratic Equations : $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$
- Complex number: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$
- The order of the matrix = m x n

  where m is the number of rows and n is the number of columns.

- Some special matrices:

  * Row matrix: one row and any number of columns
  * Column matrix: one column and any number of rows
  * Square matrix: number of rows = the number of columns
  * The Zero "Null" matrix $O_{m \times n}$: all elements are zeros
  * Diagonal matrix: It is a square matrix in which all elements are zeros except the elements of its diagonal at least one of them is not equal to zero.
  * Unit matrix: it is a diagonal matrix in which each element on the main diagonal has the numeral 1, while all other elements = 0

- Equality matrices: Two matrices A and B are equal if and only if they have the same order and the corresponding elements are equal: $a_{ij} = b_{ij}$, $\forall$ i and j

- **Matrix Transpose:**

  A of order m x n $\rightarrow A^t$ of order n x m

  Notice that: $(A^t)^t = A$ & $(AB)^t = B^t A^t$

- **Symmetric Matrices:** $A = A^t$, (A square matrix)
- **Skew Symmetric Matrices:** $A = -A^t$ (A square matrix)
- $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a\,d - b\,c$
- $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(e\,i - f\,h) - b\,(d\,i - f\,g) + c\,(d\,h - e\,g)$

Notice that: The sign of the minor determinant same as the sign of $(-1)^{i+j}$ as follow: $\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

- **Determinant of triangular Matrix:**

$$\begin{vmatrix} a & 0 & 0 \\ d & e & 0 \\ g & h & i \end{vmatrix} \text{ or } \begin{vmatrix} a & b & c \\ 0 & e & f \\ 0 & 0 & i \end{vmatrix} = a\,e\,i$$

Equals the product of the elements of its principal diagonal

- **Area of Δ ABC:** X(a , b), Y(c , d), Z(e , f) = |A| where:

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}$$

Notice that: if area = zero, then A B and C are collinear

- <u>Cramer's rule</u>

- **Solving a system of Linear equations in two variables:**

$$a\,x + b\,y = m\ ,\ c\,x + d\,y = n,$$

Let $\Delta \neq 0$, then the solution of the system is:

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix},\ \Delta x = \begin{vmatrix} m & b \\ n & d \end{vmatrix},\ \Delta y = \begin{vmatrix} a & m \\ c & n \end{vmatrix} \therefore X = \frac{\Delta x}{\Delta}\ ,\ Y = \frac{\Delta y}{\Delta}$$

$$\therefore S.S = \{(x\ ,\ y)\}$$

Solving a system of Linear equations in three variables:

$$a_1\,x + b_1\,y + c_1\,z = m,\ a_2\,x + b_2\,y + c_2\,z = n,\ a_3\,x + b_3\,y + c_3\,z = k$$

Let $\Delta \neq 0$, then the solution of the system is:

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix},\ \Delta x = \begin{vmatrix} m & b_1 & c_1 \\ n & b_2 & c_2 \\ k & b_3 & c_3 \end{vmatrix},$$

$$\Delta y = \begin{vmatrix} a_1 & m & c_1 \\ a_2 & n & c_2 \\ a_3 & k & c_3 \end{vmatrix},\ \Delta z = \begin{vmatrix} a_1 & b_1 & m \\ a_2 & b_2 & n \\ a_3 & b_3 & k \end{vmatrix},$$

$$\therefore X = \frac{\Delta x}{\Delta}\ ,\ Y = \frac{\Delta y}{\Delta}\ ,\ Z = \frac{\Delta z}{\Delta}$$

$$\therefore S.S = \{(x\ ,\ y\ ,\ z)\}$$

- **The multiplicative inverse of the matrix A:**

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ let $\Delta \neq 0$, $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then:

$$A^{-1} = \frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Notice that: $A\,A^{-1} = A^{-1}\,A = I$

- **Solving a system of Linear equations in two variables using Inverse Matrix:**

$a_1x + b_1y = k_1$, $a_2x + b_2y = k_2$ then: $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$

$$A = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix},\ X = \begin{pmatrix} x \\ y \end{pmatrix},\ C = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix} \text{ Then:}$$

$$AX = C \quad \therefore X = A^{-1}C,\ |A| \neq 0$$

Remember that (Trigonometry):

- The area of the circle is $= \pi r^2$
- The circumference of the circle is $= 2\pi r$
- The central angle: $\theta^{rad} = \dfrac{\ell}{r}$ or $\ell = \theta^{rad} \times r$

$r \rightarrow$ is the Radius of this circle

$\ell \rightarrow$ the length of the arc

- Relation between degree measure and radian measure:

$$\frac{x^{o}}{180^{o}} = \frac{\theta^{rad}}{\pi}$$

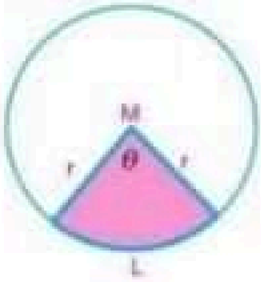
$\theta^{rad} \rightarrow$ is the radian measure

$X^{\circ} \rightarrow$ is the degree measure

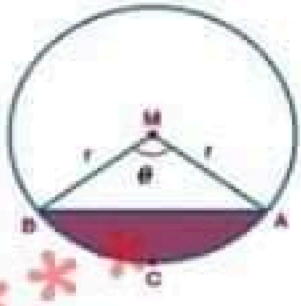
- Area of the circular sector $= \dfrac{\theta^o}{360}\pi r^2$

$$= \frac{1}{2} r^2 \theta^{rad}$$

$$= \frac{1}{2}\ell r$$



- Area of the circular segment $= \dfrac{1}{2} r^2 (\theta^{rad} - \sin\theta)$



- Area of the triangle $= \dfrac{1}{2}$ AB x BC Sin B

$$= \frac{1}{2} AB \times AC \sin A$$

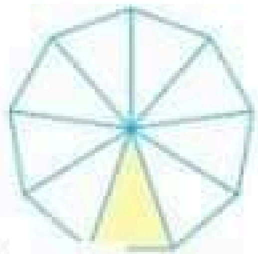
$$= \frac{1}{2} AC \times CB \sin C$$

$= \dfrac{1}{2}$ the lengths of two sides x sine the included angle between them.

- The Area of a Convex Quadrilateral $= \dfrac{1}{2} D_1 \times D_2 \sin\theta$

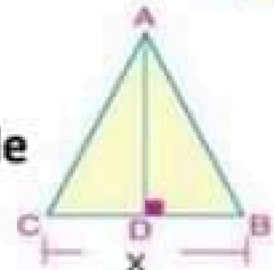
$= \dfrac{1}{2}$ lengths of its diagonals x sine the included angle between them

- The area of a regular polygon $= \dfrac{1}{4} n X^2 \cot\dfrac{\pi}{n}$

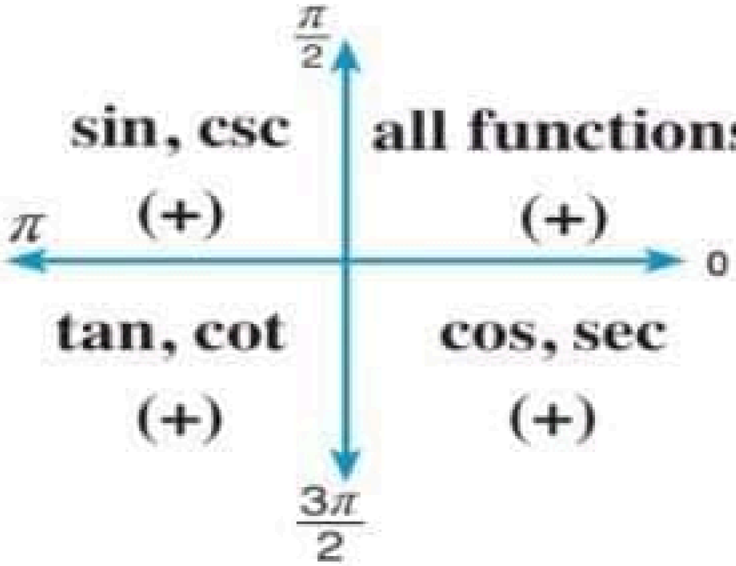


Note: $AD = \dfrac{1}{2} x \cot\dfrac{\pi}{n}$

<u>where</u>: n is number of sides , x length of its side



- **Summary of signs of all trigonometric ratios:**



- **Basic Trigonometric Identities**

$\text{Sin}^2\,\theta + \text{Cos}^2\,\theta = 1$

$\text{Sin}^2\,\theta = 1 - \text{Cos}^2\,\theta$ & $\text{Cos}^2\,\theta = 1 - \text{Sin}^2\,\theta$

$1 + \text{Tan}^2\,\theta = \text{Sec}^2\,\theta \rightarrow\rightarrow\rightarrow \text{Sec}^2\,\theta - \text{Tan}^2\,\theta = 1$

$1 + \text{Cot}^2\,\theta = \text{Csc}^2\,\theta \rightarrow\rightarrow\rightarrow \text{Csc}^2\,\theta - \text{Cot}^2\,\theta = 1$

$\text{Sin}\,2\theta = 2\,\text{Sin}\,\theta\,\text{Cos}\,\theta$

$\text{Cos}^2\,\theta - \text{Sin}^2\,\theta = 2\,\text{Cos}^2\,\theta - 1$

$= 1 - 2\,\text{Sin}^2\,\theta$

- **General solution of trigonometric equations:**

<u>When</u>: $\text{Sin}(\alpha) = \text{Cos}(\beta)$, then $\alpha \pm \beta = \frac{\pi}{2} + 2\pi\,n,\ n \in Z$

<u>When</u>: $\text{Csc}(\alpha) = \text{Sec}(\beta)$, then $\alpha \pm C = \frac{\pi}{2} + 2\pi\,n,\ \ n \in Z$

$$\alpha \neq n\,\pi,\ \beta \neq (2n+1)\frac{\pi}{2}$$

<u>When</u>: $\text{Tan}(\alpha) = \text{Cot}(\beta)$, then $\alpha + \beta = \frac{\pi}{2} + \pi\,n,\ n \in Z$,

$$\alpha \neq (2n+1)\frac{\pi}{2},\ \beta \neq n\,\pi$$

- Trigonometric functions of angles of measures

$$\theta, (180° \pm \theta)$$

$\sin(180° + \theta) = -\sin\theta$, $\sin(180° - \theta) = \sin\theta$

$\cos(180° + \theta) = -\cos\theta$, $\cos(180° - \theta) = -\cos\theta$

$\tan(180° + \theta) = \tan\theta$, $\tan(180° - \theta) = -\tan\theta$

- Trigonometric functions of angles of measures

$$\theta, (360° - \theta)$$

$\sin(360° - \theta) = -\sin\theta$, $\csc(360° - \theta) = -\csc\theta$

$\cos(360° - \theta) = \cos\theta$, $\sec(360° - \theta) = \sec\theta$

$\tan(360° - \theta) = -\tan\theta$, $\cot(360° - \theta) = -\cot\theta$

- Trigonometric functions of angles of measures

$$\theta, (90° \pm \theta)$$

$\sin(90° + \theta) = \cos\theta$, $\sin(90° - \theta) = \cos\theta$

$\cos(90° + \theta) = -\sin\theta$, $\cos(90° - \theta) = \sin\theta$

$\tan(90° + \theta) = -\cot\theta$, $\tan(90° - \theta) = \cot\theta$

- Trigonometric functions of angles of measures

$$\theta, (270° \pm \theta)$$

$\sin(270° + \theta) = -\cos\theta$, $\sin(270° - \theta) = -\cos\theta$

$\cos(270° + \theta) = \sin\theta$, $\cos(270° - \theta) = -\sin\theta$

$\tan(270° + \theta) = -\cot\theta$, $\tan(270° - \theta) = \cot\theta$

- In the function f where $f(\theta)$ = Sin θ then:

  <u>The domain</u> is $]-\infty , \infty[$

  <u>The range</u> is $[-1 , 1]$

  The cosine function is periodic with period 2π

The maximum value = 1 and takes place at the points

$$\theta = \frac{\pi}{2} \pm 2 n \pi , \quad n \in Z$$

The minimum value = – 1 and takes place at the points

$$\theta = \frac{3\pi}{2} \pm 2 n \pi , \quad n \in Z$$

- In the function f where $f(\theta)$ = Cos θ then:

  <u>The domain</u> is: $]-\infty , \infty[$

  <u>The range</u> is: $[-1 , 1]$

  The cosine function is periodic with period 2π

The maximum value = 1 and takes place at the points

$$\theta = \pm 2 n \pi , \quad n \in Z$$

The minimum value = – 1 and takes place at the points

$$\theta = \pi \pm 2 n \pi , \quad n \in Z$$

Final math revision second Term secondary1_2019

First: Complete the following questions

1) The area of Δ ABC where: A(9 , 4) , B(0 , 16) , C(0 , 0) equals..........

2) The area of Δ ABC in which: AB = 8 cm , BC = 6 cm and $m(\angle B) = 30^\circ$ equals..........

3) If: $A = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 5 \end{pmatrix}$ then $(AB)^t$ =..........

4) $\begin{vmatrix} 8 & 5 \\ 7 & 3 \end{vmatrix}$ =..............

5) If each of the matrices A and B is of order 3 x 1, then the resultant matrix of A – 2B is of order......

6) If A is a matrix of order 1 x 3, B is a matrix of order.............. then AB is a matrix of order 1 x 2.

7) If the matrix A is of order m x n, B is a matrix of order L x K, then the required condition that makes AB defined is............

8) If: $A = \begin{pmatrix} 3 & 2 & 1 \\ -1 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$

then: A – B =..................

9) $\begin{vmatrix} 3 & 0 & 0 \\ 5 & -2 & 0 \\ 4 & 1 & 2 \end{vmatrix}$ =................

10) If: A = $\begin{pmatrix} 1 & 4 & 2 \\ 0 & -6 & 8 \end{pmatrix}$ then A^t =..........

11) If the matrix $\begin{pmatrix} a & 2i \\ 2i & -1 \end{pmatrix}$ has a multiplicative inverse

then: a =............

12) If: A = $\begin{pmatrix} -1 & 2 & 5 \\ 3 & 0 & 4 \end{pmatrix}$ then A^t of order..........

13) If: $\begin{pmatrix} x^2 - 3 & 2 \\ 5 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 5 & 1 \end{pmatrix}$ Then: x =..........

14) If: $\begin{pmatrix} 15 & 5 \\ 20 & 10 \end{pmatrix} = 5\begin{pmatrix} 3a & 1 \\ 4 & 2 \end{pmatrix}$ Then: a =..........

15) If: A = $\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$, B = $\begin{pmatrix} 0 & 4 \\ 2 & 6 \end{pmatrix}$ Then: BA =..........

16) If: A = $\begin{pmatrix} 1 & 2 \\ 3 & -2 \end{pmatrix}$ Then: A^2 =................

17) $A^{-1} = \begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$ Then: A =..............

18) A = $\begin{pmatrix} -1 & 0 \\ 8 & -2 \end{pmatrix}$ Then: A^{-1} =.............

19) If: $\begin{pmatrix} x + 3 \\ y - x \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ Then: x =......... , y =.........

20) If: $\begin{pmatrix} 4 & 1 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -3 & x \end{pmatrix}$ = I Then x =..........

21) The area of circular sector whose circle radius length = 6 cm and its central angle 30° is.........

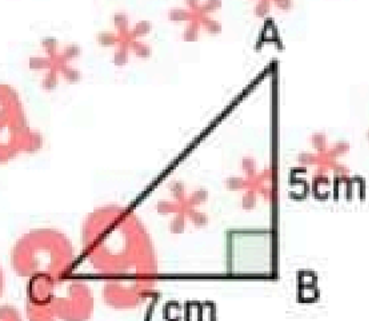
22) The area of the regular pentagon whose side length = 10 cm equals............ To nearest tenth

23) The area of the quadrilateral whose diagonals lengths 12 cm , 8 cm and the measure of the concluded angel between them = 30° is.........

24) A circular sector whose perimeter 4 r cm where r is the radius length of its circle then its central angle measure =.........

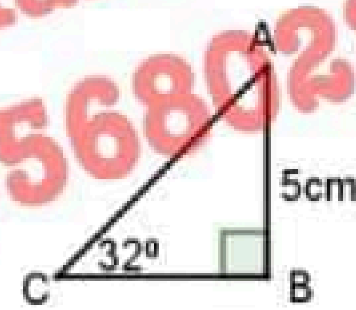
25) In the opposite figure:

m(∠C) =.............



26) In the opposite figure:

BC =....... To nearest cm



27) The area of the triangle ABC in which m(∠A) = 48° AB = 9 cm, AC = 12 cm, to the nearest hundredth.......

28) If: Sec θ + Tan θ = 4 , then: Sec θ − Tan θ =........

29) $\dfrac{Sin(\frac{\pi}{2} - \theta)}{Cos(\frac{\pi}{2} - \theta)}$ =.....................

30) The two points (4 , 3) and (3 , 2) $\in$ the S.S of the inequality $x + y$ ……… 5

31) If: $\tan^2\theta = 5$, then $\sec^2\theta =$…………

32) In Δ ABC, if AB = AC = 10 cm , $m(\angle B) = 30°$, then its area =………………… cm^2

33) The simplest form of:

$$(\cos\theta - \sin\theta)^2 + 2\sin\theta\cos\theta = \ldots\ldots\ldots\ldots$$

34) The simplest form of: $\sin^2\theta + \cos^2\theta + \cot^2\theta =$…….

35) The area of equilateral triangle of side length 10 cm equals………………

36) The general solution of: $\cos\theta = \sin\theta$ is…………

37) If: $\cot\theta = 2$, then : $\csc^2\theta =$……….

38) $\sec^2 7\theta - \tan^2 7\theta =$…………

39) If: $\sin\theta\cos\theta = \frac{1}{10}$, then: $(\sin\theta - \cos\theta)^2 =$………

40) $(\sin^2\theta + \cos^2\theta)^9 =$…………

41) $7\sin^2\theta + 7\cos^2\theta =$…………

42) If: $A - A^t = \square$ then A is called…………

43) A circular sector whose perimeter 25 cm and the length of its arc = 7 cm then its area =………

44) The area of the circular segment in which the length of the radius of its circle is 7 cm and its height 3 cm is................

45) If the matrix $\begin{pmatrix} a & 6 \\ 2 & a-1 \end{pmatrix}$ has a multiplicative inverse then: a =.....

46) If the matrix $\begin{pmatrix} x-1 & -2 \\ 1 & x-1 \end{pmatrix}$ has a multiplicative inverse then: x =.....

47) The area of the minor circular segment in which the length of its chord is 12 cm, and its height is 2 cm. to the nearest cm^2 is..............................

48) The S.S of the equation: $4 \sin^2 \theta = 3$ is............ Where $\theta \in [0, 2\pi[$

49) The S.S of the equation: $3 \sec^2 \theta = 4$ is............ Where $\theta \in [0, 2\pi[$

50) If: $5^{\sin \theta} = \frac{1}{25}$, $\theta \in [0, 2\pi]$, then the S.S =.........

51) If: $3^{\cos \theta} = 1$, $\theta \in [0, 2\pi]$, then the S.S =.........

52) The perimeter of the circular sector =..................

Second: Choose the correct answer.

1) If: $\begin{vmatrix} 2x & 2 \\ 4 & 3 \end{vmatrix} = 10$, then: x =...... (2 , 3 , 4 , 5)

2) If: A is a matrix of order 3 x 3 , then the number of elements of the matrix A is.... (3 , 6 , 9 , 12)

3) If: A is a matrix of order 2 x 3 , B^t is a matrix of order 1 x 3 then the order of the matrix AB is.......

(3 x 3 , 3 x 1 , 2 x 1 , 1 x 2)

4) If the matrix $\begin{pmatrix} a & 8 \\ 2 & a \end{pmatrix}$ has no multiplicative inverse then: a =..... (4 , ± 4 , $\in R - \{4\}$, $\in R - \{-4 , 4\}$)

5) If: $A = \begin{pmatrix} 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$ then the possible operation from the following is.......

(A + B , $A^t + B^t$, AB , $A B^t$)

6) If: $A + A^t = \square$ then A is.....

(symmetric , skew symmetric , row matrix , column matrix)

7) The S.S of the two equations: 2x – 3y = 1 , 3x + 2y =8 is..... ((3 , 2) , (1 , 2) , (2 , 1) , (2 , 3))

8) The point ....belongs to S.S of the following inequalities X > 2 , y > 1 and x + y $\geq$ 3 is......

((3 , 2) , (1 , 2) , (2 , 1) , (1 , 3))

9) The point ....belongs to S.S of the following inequalities $X \geq 0$, $y \geq 0$, $2x + y < 4$ and $x + 3y < 6$ is...... ((3 , 0) , (1 , –3) , (2 , 1) , (2 , 3))

10) The point ....belongs to S.S of the following inequality: $y < 2x + 3$ is...........

((–1 , –1) , (–1 , 1) , (–3 , –3) , (0 , 3))

11) The point ....$\notin$ the S.S of: $2x - y \leq 7$ in R x R is..... ((0 , 0) , (2 , 0) , (3 , –2) , (5 , 4))

12) The point at which the function $P = 40x + 20y$ has a maximum value is..... ((3 , 1) , (1 , 2) , (3 , 2) , (1 , 3))

13) The point at which the function $P = 35x + 10y$ has a minimum value is..........

((0 , 0) , (0 , 10) , (0 , 40) , (20 , 10))

14) If the perimeter of circular sector = 10 cm and the length of its arc = 2 cm then its area =..... cm^2

(20 , 10 , 8 , 4)

15) If the area of circular sector = 110 cm^2 and its central angle = 2.2^{rad} then the radius length of their circle =..... cm (20 , 10 , 2 , 5)

16) The area of equilateral triangle of 6 cm length equals........ cm^2 ($6\sqrt{3}$, $9\sqrt{3}$, $12\sqrt{3}$, $18\sqrt{3}$)

17) If the area of circular sector = 4 cm^2 and its arc length = 2 cm then their perimeter =..... cm

(20 , 10 , 8 , 6)

18) The S.S of the equation: Cos θ + Sin θ = 0 is.....° ,

$180° < \theta < 360°$ ({210} , {225} , {240} , {315}

19) If: $0° < \theta < 360°$, Sin θ + 1 = 0 then: θ =........ °

(0 , 90 , 180 , 270)

20) If: $0° < \theta < 180°$, $\sqrt{3}$ Tan $\theta - 1 = 0$ then: θ =..... °

(30 , 60 , 120 , 150)

21) The simplest form of: 1 + $Cot^2\,\theta$ is..............

($Sin^2\,\theta$, $Cos^2\,\theta$, $Sec^2\,\theta$, $Csc^2\,\theta$)

22) The simplest form of: $Sin^2\,\theta + Cos^2\,\theta - Csc^2\,\theta$ is......

(0 , 1 , $-Cot^2\,\theta$, $Tan^2\,\theta$)

23) The simplest form of: Sin(90 – θ) Csc(180 – θ)

is......... (–1 , 1 , Cot θ , Tan θ)

24) The general solution of the equation: Cos θ = 1 is......

($n\pi$, $2n\pi$, $\frac{\pi}{2} + n\pi$, $\frac{\pi}{2} + 2n\pi$)

25) The general solution of the equation: $\sin\theta - 1 = 0$ is...... $(\pi + 2\pi n \ , \ 2\pi n \ , \ \frac{\pi}{2} + n\pi \ , \ \frac{\pi}{2} + 2\pi n)$

26) If: $\sin\theta = \frac{1}{2}, \theta \in]\frac{\pi}{2}, \pi[$ then: $\theta =$.........

$(\frac{\pi}{6} + 2\pi \ , \frac{\pi}{3} + 2\pi \ , \ \frac{-\pi}{6} + \pi \ , \ \frac{-\pi}{6} + 2\pi)$

27) The general solution of the equation: $\tan\theta = \sqrt{3}$ is...... $(\frac{\pi}{3} + n\pi \ , \ \frac{\pi}{3} + 2n\pi \ , \ \frac{4\pi}{3} + 2n\pi \ , \ \frac{\pi}{6} + n\pi)$

28) $\frac{\tan\theta \cot\theta}{\sec\theta} =$.... $(\sin\theta \ , \ \cos\theta \ , \ \sec\theta \ , \ \csc\theta)$

29) The perimeter of the circular sector =........

$(\frac{1}{2}r + \ell \ , \frac{1}{2}r\ell \ , \ 2r + \ell \ , \ r + \ell)$

30) $\sin^2 2\theta + \cos^2 2\theta =$..... (1 , −1 , 4 , 2)

31) $\begin{vmatrix} x & 2 \\ 4 & x \end{vmatrix} = \begin{vmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{vmatrix}$, then: x =.........

(3 , −3 , ±3 , 9)

32) If: $AB = \begin{pmatrix} 4 & 5 \\ -1 & 3 \end{pmatrix}$, then: $B^t A^t =$.........

$(\begin{pmatrix} -1 & 4 \\ 3 & 5 \end{pmatrix} , \begin{pmatrix} 4 & 5 \\ -1 & 3 \end{pmatrix} , \begin{pmatrix} 4 & -1 \\ 5 & 3 \end{pmatrix} , \begin{pmatrix} 4 & 3 \\ 5 & 1 \end{pmatrix})$

33) If: $\begin{vmatrix} 2 - x & 2 \\ -3 & x + 2 \end{vmatrix} = 1$, then: x =................

(3 , − 3 , ±3 , ±4)

34) $(\text{Cos}\,\theta + \text{Sin}\,\theta)^2 - 2\text{Sin}\,\theta\,\text{Cos}\,\theta$ =...... (1 , 2 , 3 , 0)

35) If: $\text{Sec}\,\theta - \text{Tan}\,\theta = \frac{2}{5}$, then: $\text{Sec}\,\theta + \text{Tan}\,\theta$ =........

($\frac{2}{5}$, $\frac{5}{2}$, $-\frac{2}{5}$, $-\frac{5}{2}$)

36) The simplest form of: $\text{Sin}^2\,\theta + \text{Tan}^2\,\theta + \text{Cos}^2\,\theta$

is.......... ($\text{Sec}^2\,\theta$, 1 , $\text{Csc}^2\,\theta$, $\text{Tan}^2\,\theta$)

37) If: $0° < \theta < 180°$, $2\text{Cos}\,\theta + 1 = 0$ then: θ =..... °

(300 , 240 , 210 , 120)

38) The S.S of: $-1 \leq -x \leq 1$ in R is..........

($]-1, 1]$, $R -]-1, 1]$, $\{0, 1\}$, $[-1, 1]$)

39) $\frac{1 - Sin^2\,\theta}{1 - Cos^2\,\theta}$ =...... (1 , −1 , $\text{Tan}^2\,\theta$, $\text{Cot}^2\,\theta$)

40) The area of the equilateral triangle whose side length is x cm. equals........... cm^2

(x^2 , $\frac{\sqrt{3}}{2}x^2$, $\frac{\sqrt{3}}{4}x^2$, $\frac{1}{2}x^2$)

41) The area of the square whose side length is x cm.

equals............ cm^2 (x^2 , $\sqrt{2}\,x^2$, $\frac{\sqrt{2}}{2}x^2$, $\frac{1}{2}x^2$)

42) The area of the regular octagon whose side length is x cm. equals........

($2x^2\,Cot\,45$, $2x^2 Tan\,45$, $8\,x^2 Cot\,22.5$, $2\,x^2 Cot\,22.5$)

Third: answer the following questions:

1) If: $A = (A_{xy})\ \forall\ x, y \in \{1, 2, 3\}$ write the matrix A given that: $A_{xy} = y - x$ then find A^t

2) Find the area of Δ ABC where: $A(-4, 2)$, $B(3, 1)$, $C(-2, 5)$ using determinants

3) Find the value of x if: $\begin{vmatrix} 1 & 0 & 0 \\ 1 & x & x \\ 5 & 2 & x \end{vmatrix} = 3$

4) Find the value of x if: $\begin{vmatrix} x & 0 & 0 \\ 4 & x & x \\ 3 & 2 & x \end{vmatrix} = 3x$

5) Is the matrix $A = \begin{pmatrix} 1 & -1 & 4 \\ -1 & 2 & 6 \\ 4 & 6 & 5 \end{pmatrix}$ Symmetric or skew symmetric?

6) Is the matrix $A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 3 \\ 1 & -3 & 0 \end{pmatrix}$ Symmetric or skew symmetric?

7) If: $A = \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2d & -1 \\ 3e & 4 \end{pmatrix}$ where $A = B^t$ then find d and e

8) Find the S.S of the following equations using Cramer's method: $2x - 3y = 3$, $x + 2y = 5$

9) If: A = $\begin{pmatrix} 0 & 3x & 7 \\ x+3 & 0 & -2Z \\ 3y-x & 6 & 0 \end{pmatrix}$ is a skew symmetric matrix. Find the value of x , y , z

10) If: A = $\begin{pmatrix} 5 & 2x & 8 \\ -4 & -3 & 6 \\ x+2y & 6 & 4 \end{pmatrix}$ is a symmetric matrix.

Find the value of x , y

11) Solve the S.S of the following equations using Cramer's method: $2x + y - 2Z = 10$, $3x + 2y + 2Z = 1$

$5x + 4y + 3Z = 4$

12) Using determinants to prove that the points A(3 , 5) , B(4 , − 1) , C(5 , − 7) are collinear

13) Find the area of Δ ABC where: A(−1 , −3) , B(2 , 4) , C(−3 , 5) using determinants

14) Find a , b and c if: $\begin{pmatrix} -2 & 3 \\ 4 & -1 \end{pmatrix} + \begin{pmatrix} a & b \\ c & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$

15) If: A = $\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$ Find: $3A^{-1} + 5I$

16) If: $X^t + \begin{pmatrix} 2 & 1 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ find the matrix X

17) If: A = $\begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix}$ prove that: $A^2 - 2A - 3I = \square$

18) If: $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$ then find X where

$2X^t - AB = \begin{pmatrix} 2 & -1 \\ -5 & 2 \end{pmatrix}$

19) If: $A = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$ then find $A^t B$

20) If: $A^t = \begin{pmatrix} 2 & -4 \\ 4 & 3 \end{pmatrix}$ prove that: $A^2 - 5A + 2I = \square$

21) Find the values of a , which make the matrix $\begin{pmatrix} a & 2 \\ 8 & a \end{pmatrix}$

has a multiplicative inverse.

22) Find the values of x , which make the matrix $\begin{pmatrix} x & 9 \\ 4 & x \end{pmatrix}$

has no multiplicative inverse.

23) If: $A = \begin{pmatrix} 2 & 0 \\ 3 & 4 \end{pmatrix}$, Prove that for the matrix A, there is

a multiplicative inverse, then find it

24) Find a and b if: $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} a & 7 \\ 3 & b \end{pmatrix} = \begin{pmatrix} 7 & 11 \\ 8 & 18 \end{pmatrix}^t$

25) If: $A = \begin{pmatrix} 1 & 4 \\ 3 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$ then: find AB

26) Solve the system of the following equations using

the matrices: $3x + y = 2$, $5x + 4y = -6$

27) Solve the system of the following equations using the matrices: $3x + 2y = 5$, $2x + y = 3$

28) Use the matrices to find the two numbers in which their sum equal 10 and the difference between them equal 4.

29) Represent graphically the S.S of the following inequality $2x - 5y \geq 10$ in $\mathbb{R} \times \mathbb{R}$

30) Solve the following linear inequalities graphically: $3x + 5y \geq 15$, $y < x - 1$

31) Solve the following linear inequalities graphically: $x \geq 0$, $y \leq 2$, $2x + 3y \leq 12$

32) Use the linear programing, find each of the minimum value and the maximum value for the function $P = 4x + y$ under restrictions: $x \geq 0$, $y \geq 0$, $x + y \leq 6$, $2x + y \geq 10$

33) Use the linear programing, find each of the minimum value and the maximum value for the function $P = 3x + 2y$ under restrictions $x \geq 0$, $y \geq 0$, $x + y \leq 8$, $y \geq 3$

34) Find the greatest possible value of the function $P = 3x + 2y$ under the following restrictions:
$x \geq 0$, $y \geq 0$, $2x + y \leq 8$, $2x + 3y \leq 12$

35) Find the maximum value of the object function $P = 2x + y$ given that: $x \geq 0$, $y \geq 0$, $2x + 3y \leq 18$, $-4x + y \geq -8$

36) Find the minimum value of the object function $P = 3x + 2y$ given that: $x \geq 0$, $y \geq 0$, $2x + y \geq 8$, $x + 3y \geq 9$

37) Find the area of the circular sector whose perimeter equals 28 cm, and the length of the radius of its circle equals 8 cm.

38) A circular sector in which the measure of its angle equals 60° and the length of the radius of its circle equals 12 cm. Find its area to the nearest tenth.

39) Find the area of the regular octagon in which the length of its side equals 6 cm approximating the result to the nearest hundredth

40) Prove that: $\frac{\cos^2\theta}{1 - \sin\theta} = 1 + \sin\theta$

41) Prove that: $\frac{\cot\theta}{1 + \cot^2\theta} = \sin\theta \cos\theta$

42) Prove that: $\frac{1 + \tan^2\theta}{1 + \cot^2\theta} = \tan^2\theta$

43) Prove that: $\tan\theta + \cot\theta = \sec\theta \csc\theta$

44) Prove that: $\sin\theta \sin(90 - \theta) \tan\theta = 1 - \cos^2\theta$

45) Find the general solution: $\cos(\frac{\pi}{2} - \theta) = \frac{1}{2}$

46) Prove that: $\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos^2\theta + \cos\theta\sin^2\theta} = \csc\theta - \sec\theta$

47) Prove that: $\frac{1}{1+\cot\theta} = \frac{\tan\theta}{1+\tan\theta}$

48) Prove that: $\frac{1+\tan^2\theta}{\sec^4\theta} = 1 - \sin^2\theta$

49) If: $\frac{3\cos\theta - 2\sin\theta}{3\cos\theta + 2\sin\theta} = \frac{1}{2}$ then find $\tan\theta$

50) Find the general solution: $\cos\theta = \frac{\sqrt{2}}{2}$

51) Find the general solution: $\tan\theta = \sqrt{3}$

52) Find the general solution: $\sqrt{2}\sin\theta\cos\theta - \sin\theta = 0$

53) Solve the equation: $\sin\theta\cos\theta = \frac{1}{2}\cos\theta$ where

$0^\circ < \theta < 180^\circ$

54) Find the general solution: $\cos\theta = \sin 2\theta$

55) Find the general solution: $2\sin^2\theta = \sin\theta$

56) If $0^\circ < \theta < 360^\circ$ Find the solution set of equations:

$4\sin^2\theta - 3\sin\theta\cos\theta = 0$

57) Area of a circular sector is 270 cm^2 and the length of the radius of its circle equals 15 cm, find in radian the measure of its angle.

58) Find the S.S: $2\cos^2\theta - \cos\theta - 1 = 0$ where $\theta \in [0, 2\pi[$

59) Find the area of the circular sector in which the length of the radius of its circle is 10 cm and the measure of its angle is 1.2^{rad}

60) Find the area of the circular segment whose length of the radius of its circle equals 8 cm, and the measure of its central angle equals $150°$.

61) Find the area of the circular segment whose length of the radius of its circle equals 10 cm and the measure of its circle equals $2.\ 2^{rad}$ approximating the result the nearest hundredth.

62) Find the area of the circular segment in which the length of the radius of its circle is 8 cm and its height 4 cm

63) Circular segment in which its central angle $90°$ and its area = 56 cm^2. Find the length of its radius.

64) A chord of length 8 cm, in a circle is at a distance 3 cm from its center. Find the area of its circular segment

65) A circular sector whose perimeter equals 24 cm, and the length of its arc equals 10 cm. Find the area of its circle.

66) Circular segment in which the length of its chord = the length of its radius = 8 cm. Find its area

67) Solve the triangle ABC, right angled at B in which BC = 5 cm , AC = 13 cm

68) Solve the triangle ABC, right angled at B in each of the following cases:

A) AB = 8 cm , m(∠C) = 34°

B) AC = 26 cm , m(∠ A) = 53° 12'

69) A circle of radius 7 cm, a chord was drawn in it opposite to a central angle of measure 110°. Calculate the length of this chord.

70) An equilateral triangle whose area = $9\sqrt{3}$ cm², find the length of its side.

71) Using determinants to prove that the points A(3 , 5) , B(4 , 6) , C(5 , 7) are collinear

72) A person observed the top of a hill 2.56 km from the point on the ground. He found its depression angle was 63°. Find the distance between the top and the observer to the nearest metre

73) ABC in which: AB = 8 cm , BC = 7 cm , AC = 11 cm, find its area

74) From the top of a tower 60 metres high, the angle of depression of a body located in a horizontal level which passes through the base of the tower equals $28° 36'$. Find how far was the body from the base of the tower to the nearest metres.

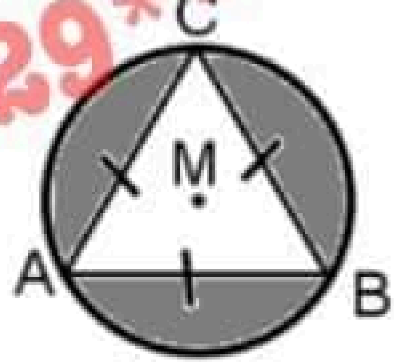
75) A person stands at 50 metres from the base of a tower. He observed the elevation angle of the top of the tower and found it to be $19° 24'$. Find the height of the tower to the nearest metre.

76) A boat was observed from the top of the lighthouse of height 50 m. it was found that its depression angle $35°$. Find the distance between the boat and the top of the lighthouse.

77) In the opposite figure:

ABC is an equilateral triangle drawn in circle M of radius equal 8 cm

find the area of each shaded circular segment



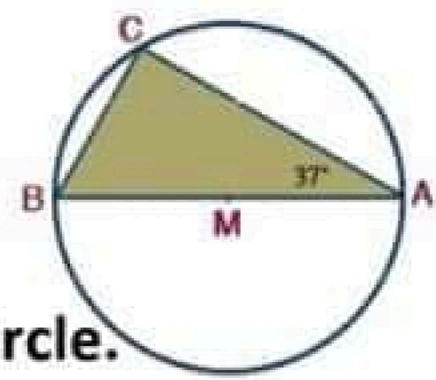
78) In the figure opposite:

circle M, $\overline{AB}$ is a diameter in it ,

AC = 12 cm, m(∠ A) = 37°

find the length of the radius of the circle.

to the nearest hundredth

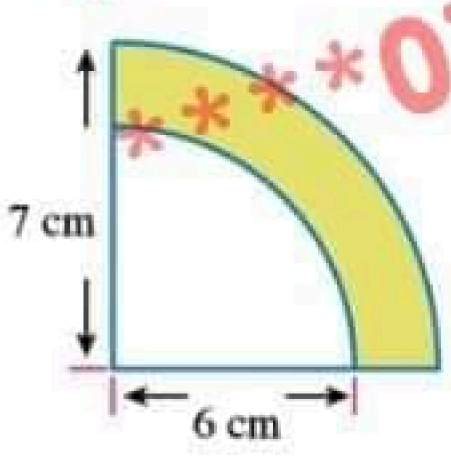


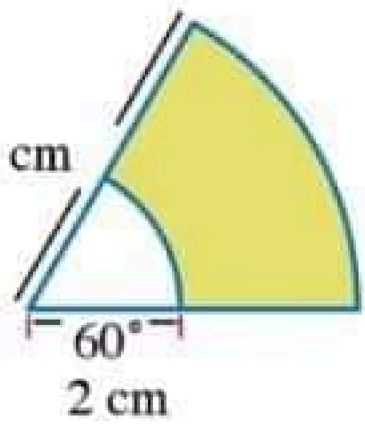
79) The curve whose equation: $y = a x^2 + b x$ passes through the two points $(3, 0)$ and $(4, 8)$ use the matrix to find the constants a and b

80) The straight line whose equation: $y + a x = c$ passes through the two points $(1, 5)$ and $(2, 1)$ use the matrix to find the constants a and b

81) ABC is an equilateral triangle of side length 24 cm drawn in a circle, find the radius length of the circle and the area of circular segment of the chord $\overline{BC}$

82) Find in terms of π the area of the shaded part in each of the following figures:





With my best wishes

Real life applications of linear programing

1) A small factory produces metal furniture 20 cupboard weekly at most of two different kinds A and B. If the profit from kind "A' is 80 pounds, and profit from kind B is 100 pounds. The factory sells from kind A at least 3 times what it sells from the second kind. Find number of cupboard from each kind to satisfy greatest possible profit to the factory.

2) Consumer: Two package of food substances, the first gives 3 calories and has 5 units of vitamin "C", the second gives 6 calories and has 2 units of vitamin "C', given that we need at least 36 calories and at least 25 units of vitamin "C". The price of the unit of the first article is 6 pounds, and of the second is 8 pounds. Find the number of each article that should be bought to obtain what we need at the least cost.

" اللهم صل وسلم وبارك على سيدنا محمد وعلى آله وصحبه وسلم "